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VAN VLECK REVISITED-THE RCS (RADAR CROSS SECTION) OF THIN WIRES

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ABSTRACT

The approximate theory of radar reflection from thin wires by Van Vleck, et. al., gives very good average radar cross section (RCS) results and good angular RCS results except end-on. In this paper, the nature of this end-on discrepancy is examined. It is found that, if the complete expressions derived by Van Vleck, et. al., are [illegible] ~~utilized~~ without the approximations made to simplify calculation of the average RCS over all angles of incidence, then very accurate RCS results are predicted for nearly all angles of incidence and for all the wire length-to-wavelength ratios between 0.45 and 50. Comparisons with the numerical results of a source distribution technique (SDT) computer program and with the results due to Ufimtsev are shown.

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FIGURES

FIGURES (Continued)

TABLE

I. INTRODUCTION

In a previous report (Ref. 1), predictions of the radar cross section (RCS) of long, thin wires by several authors were compared with experimental data. It was found that the predictions of Van Vleck, et al., (Ref. 2) were of greater validity than had previously been believed; however, the RCS of thin wires for end-on incidence was considerably in error. This error was believed to be due to the approximations made by Van Vleck, et al., to simplify the calculation of the average RCS over all angles of incidence.

To confirm this belief, the theory of Van Vleck, et al., is examined in detail in Sec. 2 of this report. It is shown that, without some of these simplifying approximations discussed above, the general theory derived by Van Vleck, et al., does indeed give a cross section that goes to zero at end-on incidence. The general theory is used to calculate the RCS of thin wires of various lengths. These results are compared with data generated by BRACT*, a source distribution technique (SDT) computer program, to verify the fact that use of the general theory instead of the approximate theory (Ref. 2) has not degraded the overall angular RCS results.

The approximate theory of Van Vleck, et al., is also compared directly with the general theory and with BRACT to determine the differences among the results. Note that results obtained using the theory of Ufimtsev (Refs. 1 and 3) have also been compared with the BRACT results. These comparisons are presented in Sec. 3 of this report. A brief discussion follows (Sec. 4).

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*The BRACT computer program, which solves the thin wire integral equation of the complete electromagnetic scattering problem for arbitrary wire structures, has been validated through extensive use and is considered to have an accuracy of better than 1 dB (Refs. 1 and 4).

II. THEORY

Consider a plane wave incident on a thin wire of length 2ℓ, as shown in Fig. 1.

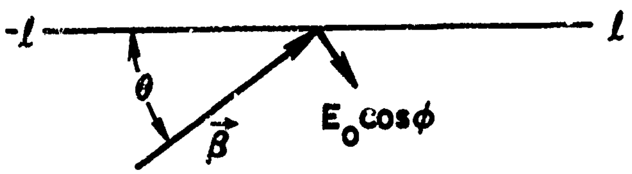

Fig. 1. Plane of Incidence

The angle of incidence is θ, and the angle between $\vec{E}_0$ (the electric field vector) and the plane formed by the wire and the propagation vector $\vec{\beta}$ is ϕ. The CGS system of units is used throughout this paper. Then, for the scattered field ($\vec{E}^s$), from Maxwell's homogeneous equations

$$\vec{E}^s = -\vec{\nabla}\phi^s - \frac{1}{c}\frac{\partial\vec{A}^s}{\partial t} \tag{1}$$

where $\vec{A}^s$ is the scattered vector potential and ϕ^s is the scattered scalar potential. If the Lorentz gauge is used

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0 \tag{2}$$

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Eq. (1) becomes

$$\frac{\partial \vec{E}^s}{\partial t} = c\vec{\nabla}\vec{\nabla}\cdot\vec{A}^s - \frac{1}{c}\frac{\partial^2 \vec{A}}{\partial t^2} \tag{3}$$

If all time dependence appears at a single frequency as $e^{-i\omega t}$, then Eq. (3) becomes

$$-i\omega\vec{E}^s = c\vec{\nabla}\vec{\nabla}\cdot\vec{A} + \frac{\omega^2}{c}\vec{A} \tag{4}$$

Now, if only the tangential component of the field at the surface of the wire is considered and if it is assumed that the wire is a perfect conductor lying on the z axis, $E_z^s = - E_z^i$ ($\vec{E}^i$ is the incident field) and Eq. (4) becomes

$$\frac{\partial^2 A_z}{\partial^2 z} + \beta^2 A_z = i\beta E_z^i \tag{5}$$

where the superscript s has been dropped. The incident field is given by

$$\vec{E}_o \, e^{i\vec{\beta}\cdot\vec{X} - i\omega t}$$

If the time component is neglected (Fig. 1)

$$E_z^i = E_o \cos\phi \sin\theta \, e^{i\beta z\cos\theta} \tag{6}$$

or

$$\frac{\partial^2 A_z}{\partial^2 z} + \beta^2 A_z = i\beta E_o \cos\phi \sin\theta \, e^{i\beta z \cos\theta} \tag{7}$$

The homogeneous solution of Eq. (7) is

$$A \cos \beta z + B \sin \beta z \tag{8}$$

and the inhomogeneous solution is

$$iE_o \cos\phi \sin\theta \int_o^z e^{i\beta\xi\cos\theta} \sin\beta(z-\xi)\, d\xi$$

$$= \frac{iE_o \cos\phi}{\beta \sin\theta} \left(e^{iqz} - \cos\beta z - i\cos\theta \sin\beta z\right) \tag{9}$$

where $q = \beta \cos\theta = 2\pi/\lambda \cos\theta$.

Recall from Maxwell's inhomogeneous equations that the vector potential due to a current distribution is given by

$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial \vec{A}^2}{\partial t^2} = -\frac{4\pi}{c} \vec{J} \tag{10}$$

which has a general free space solution

$$\vec{A} = \frac{1}{c} \int \frac{\vec{J}\, \delta[t' + (|\overline{x} - \overline{x}'|/c) - t]}{|\overline{x} - \overline{x}'|} dt' \; dx'^3 \tag{11}$$

If $J = J(\vec{X})\,e^{-i\omega t}$ and if it is assumed that the current flows only along the z′ direction and resides at the axis of the wire, while the vector potential is wanted at the surface of the wire

$$A_z(z) = \frac{\pi a^2}{c}\int_{-\ell}^{\ell} \frac{J\,e^{-i\beta r}}{r}\,dz' \qquad \text{and} \qquad r = \sqrt{(z - z')^2 + a^2} \tag{12}$$

Or, if $J = I/\pi a^2$ and if the vector potential solutions of Eqs. (7) and (12) are equated

$$\int_{-\ell}^{\ell} \frac{I(z')\,e^{-i\beta r}}{r}\,dz' = A_1 \cos\beta z + B_1 \sin\beta z + \frac{i\omega E_o \cos\phi}{\beta^2 \sin\theta}\,e^{iqz} \tag{13}$$

This is identical to Eq. (1) in Ref. 2; note that the coefficients of the sine and cosine terms of Eqs. (8) and (9) have been combined. Equation (13) is an integral equation that must be solved for the current on the wire, subject to the boundary condition that the current vanish at $\pm\ell$. Further, the current should vanish for $\theta = 0$, i.e., the right-hand side of Eq. (13) should vanish at $\theta = 0$.

In order to solve for Eq. (13), assume that the current I(z) is given by

$$I(z) = \alpha\,e^{iqz} + Y_1 \cos\beta z + iY_2 \sin\beta z \tag{14}$$

where α, Y_1, and Y_2 do not depend on z. Note that Eq. (14) is just an approximation to the current and that a more accurate expression would require an iterative solution (to be discussed below).

The value of α is determined by substituting the e^{iqz} term of Eq. (14) into Eq. (13) and by equating with the e^{iqz} terms

$$\alpha \int_{-\ell}^{\ell} \frac{e^{iqz'} e^{-i\beta r}}{r} dz' = \frac{i\omega E_o \cos \phi}{\beta^2 \sin \theta} e^{iqz} \tag{15}$$

The integral on the left-hand side is broken into three terms for easy evaluation (Ref. 5)

$$\int_{-\ell}^{\ell} \frac{e^{iqz'} e^{-i\beta z}}{r} dz' \equiv e^{iqz} \int_{-\ell}^{\ell} \frac{\cos \beta r}{r} dz' + \int_{-\ell}^{\ell} \frac{(e^{iqz'} - e^{-iqz})}{r} \cos \beta r \, dz' - i \int_{-\ell}^{\ell} \frac{e^{iqz'} \sin \beta r}{r} dz' \tag{16}$$

Let

$$\int_{-\ell}^{\ell} \frac{\cos \beta r}{r} dz' = Z(z) = \int_{-\ell}^{\ell} \frac{1}{r} dz - \int_{-\ell}^{\ell} \frac{(1 - \cos \beta r)}{r} dz \tag{17}$$

The second term on the right-hand side of Eq. (17) will not go to zero for $r = 0$; therefore, we may replace r by $|z - z'|$ for this term and for the two right-hand terms in Eq. (16). By a change of variables in Eqs. (16) and (17) and by use of the sine and cosine integrals

$$\text{Cin } x = \int_0^x \frac{1 - \cos t}{t} dt$$

$$\text{Si } x = \int_0^x \frac{\sin t}{t} dt$$

Equations (16) and (17) are evaluated as

$$Z(z) = \log\left[\frac{[(\ell + z)^2 + a^2]^{1/2} + (\ell + z)}{[(\ell - z)^2 + a^2]^{1/2} - (\ell - z)}\right] - \mathrm{Cin}\,\beta(\ell + z) - \mathrm{Cin}\,\beta(\ell - z) \tag{18}$$

$$\begin{aligned}
K(z)\, e^{iqz} &= \int r^{-1}\, e^{iqz'}\, e^{-i\beta r}\, dz' \\
&= \frac{1}{2} e^{-iqz}\{2Z(z) + 2\mathrm{Cin}\,\beta(\ell - z) + 2\mathrm{Cin}\,\beta(\ell + z) - \mathrm{Cin}(\beta + q)(\ell - z) - \mathrm{Cin}(\beta - q)(\ell + z) \\
&\qquad - \mathrm{Cin}(\beta - q)(\ell - z) - \mathrm{Cin}(\beta + q)(\ell + z) \\
&\qquad - i[\mathrm{Si}(\beta + q)(\ell - z) + \mathrm{Si}(\beta - q)(\ell + z) + \mathrm{Si}(\beta - q)(\ell - z) + \mathrm{Si}(\beta + q)(\ell + z)]\} \\
&\quad + \frac{1}{2} e^{iqz}\{\mathrm{Cin}(\beta + q)(\ell - z) + \mathrm{Cin}(\beta - q)(\ell + z) - \mathrm{Cin}(\beta + q)(\ell + z) - \mathrm{Cin}(\beta - q)(\ell - z) \\
&\qquad + i[\mathrm{Si}(\beta + q)(\ell - z) + \mathrm{Si}(\beta - q)(\ell + z) - \mathrm{Si}(\beta + q)(\ell + z) - \mathrm{Si}(\beta - q)(\ell - z)]\}
\end{aligned}$$

Since

$$\alpha K(z)\, e^{iqz} = \frac{i\omega \cos\phi\, E_o\, e^{iqz}}{\beta^2 \sin\theta} \tag{19}$$

and since α should be independent of z, an average over the wire is taken

$$\overline{Z}(z) = 2\left[\log(2\ell/a) + \log 2 + a/2\ell - \mathrm{Cin}\,2\beta\ell - \frac{\sin 2\beta\ell}{2\beta\ell}\right]$$

$$\begin{aligned}
\overline{K}(z) &= 2\log(2\ell/a) + 2\log 2 + a/\ell - \mathrm{Cin}\,2(\beta + q)\ell - \mathrm{Cin}\,2(\beta - q)\ell - \frac{\sin 2(\beta + q)\ell}{2(\beta + q)\ell} - \frac{\sin 2(\beta - q)\ell}{2(\beta - q)\ell} \\
&\quad - i\left[\mathrm{Si}\,2(\beta + q)\ell + \mathrm{Si}\,2(\beta - q)\ell + \frac{\cos 2(\beta + q)\ell - 1}{2(\beta + q)\ell} + \frac{\cos 2(\beta - q)\ell - 1}{2(\beta - q)\ell}\right]
\end{aligned} \tag{20}$$

or

$$\alpha = \frac{i\omega \cos\phi\, E_o}{\overline{K}\beta^2 \sin\theta} \tag{21}$$

Similarly, Υ_1 and Υ_2 are related to A_1 and B_1 by

$$A_1 = L\Upsilon_1 \quad \text{and} \quad B_1 = iL\Upsilon_2 \tag{22}$$

where

$$L = 2\log(2\ell/a) + 2\log 2 + a/\ell - \mathrm{Cin}\, 4\beta\ell - \frac{\sin 4\beta\ell}{4\beta\ell} - 1$$

$$- i\left(\mathrm{Si}\, 4\beta\ell + \frac{\cos 4\beta\ell - 1}{4\beta\ell}\right) \tag{23}$$

Note that when q goes to β or when θ goes to zero, the value of $\overline{K}$ approaches the value of L. The use of the complete expressions for $\overline{K}$ and L differs from Van Vleck's usage in that he uses the asymptotic values of the Cin and Si functions for large argument. The usage is incorrect when $\beta \pm q \rightarrow 0$.

In order to determine A_1 and B_1 or Υ_1 and Υ_2, the boundary conditions on the current are applied. However, if they are applied directly to Eq. (14), the results would give an infinite current for $\beta = \pm n\pi/2$; n = 1, 2, 3 $\cdots$ and is identically zero for $q = \pm\beta$. Instead, the integral in Eq. (13) is broken into three terms, as in Eq. (16)

$$I(z)\, Z(z) = -\int_{-\ell}^{\ell} r^{-1}\left[I(z') - I(z)\right]\cos\beta r\, dz' + i\int_{-\ell}^{\ell} r^{-1}\, I(z')\sin\beta r\, dz'$$

$$+ \left(\frac{i\omega\cos\phi}{\beta^2\sin\theta}\right) E_o\, e^{iqz} + A_1\cos\beta z + B_1\sin\beta z \tag{24}$$

It has previously been mentioned that a more accurate representation of the current would require an iterative solution. Equation (24) would be the basis for such a solution, with the previous solution (e.g., Eq. 14) for the current used in the right-hand side of Eq. (24). The averaged value of $Z(z)$ would be used in performing any integrations past the first iteration to the current. Instead of iterating, Eq. (24) is forced to obey the boundary conditions $I(\pm\ell) = 0$ by using the zeroth approximation to the current (Eq. 14) in the right-hand side of Eq. (24) to determine γ_1 and γ_2. This leads to Eqs. (25) and (26)

$$Q\, e^{iq\ell} + A_1 \cos\beta\ell + B_1 \sin\beta\ell = \alpha D\, e^{iq\ell} + (\gamma_1/2)\left(E\, e^{i\beta\ell} + F\, e^{-i\beta\ell}\right) + (\gamma_2/2)\left(E\, e^{i\beta\ell} - F\, e^{-i\beta\ell}\right) \quad (25)$$

$$Q\, e^{-iq\ell} + A_1 \cos\beta\ell - B_1 \sin\beta\ell = \alpha G\, e^{iq\ell} + (\gamma_1/2)\left(E\, e^{i\beta\ell} + F\, e^{-i\beta\ell}\right) - (\gamma_2/2)\left(E\, e^{i\beta\ell} - F\, e^{-i\beta\ell}\right) \quad (26)$$

where

$$Q = \frac{i\,\omega \cos\phi\, E_o}{\beta^2 \sin\theta} \quad (27)$$

and

$$\begin{aligned} D &= \mathrm{Cin}\, 2\beta\ell - \mathrm{Cin}\, 2(\beta + q)\ell - i\, \mathrm{Si}\, 2(\beta + q)\ell \\ G &= \mathrm{Cin}\, 2\beta\ell - \mathrm{Cin}\, 2(\beta - q)\ell - i\, \mathrm{Si}\, 2(\beta - q)\ell \\ E &= \mathrm{Cin}\, 2\beta\ell - \mathrm{Cin}\, 4\beta\ell - i\, \mathrm{Si}\, 4\beta\ell \\ F &= \mathrm{Cin}\, 2\beta\ell \end{aligned} \quad (28)$$

From Eq. (22), the following solutions are obtained for γ_1 and γ_2

$$\gamma_1 = - \frac{Q}{\overline{\overline{K}}} T_1$$

$$\gamma_2 = - \frac{Q}{\overline{\overline{K}}} T_2$$

and the current is

$$I(z) = \frac{Q}{\overline{\overline{K}}} \left[e^{iqz} - T_1 \cos \beta z - iT_2 \sin \beta z \right] \tag{29}$$

where

$$T_1 = \frac{[2\overline{K} \cos q\ell - (D\, e^{iq\ell} + G\, e^{-iq\ell})]}{[2L \cos \beta\ell - (E\, e^{i\beta\ell} + F\, e^{-i\beta\ell})]} \tag{30}$$

$$T_2 = \frac{[2\, i\overline{K} \sin q\ell - (D\, e^{iq\ell} - G\, e^{-iq\ell})]}{[2\, iL \sin \beta\ell - (E\, e^{i\beta\ell} - F\, e^{-i\beta\ell})]} \tag{31}$$

The current thus correctly goes to zero for $q = \pm\beta$.

The vector potential in the far field is then given by

$$\vec{A}(\vec{X}) \cong \frac{1}{c} \frac{e^{i\beta r}}{r} \left(\int_{-\ell}^{\ell} I(z')\, e^{iq'z'}\, dz' \right) \vec{k} \tag{32}$$

where $\vec{k}$ is the unit vector in the z direction and $q' = \beta \cos \theta'$, with θ' the received angle.

The far field scattered $\vec{E}$ (from Maxwell's equation) is written as

$$\vec{E} = \frac{i}{\beta} \vec{\nabla} \times \vec{\nabla} \times \vec{A} \cong \frac{i\beta}{c} \frac{e^{i\beta r}}{r} \sin\theta' \int_{-\ell}^{\ell} I(z')\, e^{iq'z'} dz' \tag{33}$$

Assume that the monostatic cross section is desired and that the polarization angle of the detector is the same as that of the transmitter. The scattered field detected is then

$$E = \frac{-E_o\, e^{i\beta r}}{\beta r \bar{K}} \cos^2\phi \left[\frac{\sin 2q\ell}{\cos\theta} - [T_1 + T_2] \frac{\sin(\beta + q)\ell}{1 + \cos\theta} - [T_1 - T_2] \frac{\sin(\beta - q)\ell}{1 - \cos\theta} \right] \tag{34}$$

The RCS of the wire is defined as

$$\sigma(\theta, \phi) = \frac{4\pi r^2 E^2}{E_o^2} = \frac{\lambda^2 \cos^4\phi}{\pi \bar{K}\, \bar{K}^*} E_1\, E_1^* \tag{35}$$

where E_1 is the quantity in brackets in Eq. (34).

III. COMPARISON OF RESULTS

The expressions given in the previous section have been programmed, and calculations have been performed for the cases listed in Table 1.

Table 1. Thin Wire Parameters

Case No.	Wavelength, m	Polarization[a]		βa	$\beta \ell$
		Transmitted	Received		
1	1.00	Linear	Linear	3.14×10^{-2}	1.415
2	0.227	Circular	Circular	4.2×10^{-3}	4.44
3	1.00	Circular	Circular	3.95×10^{-2}	17
4	0.69	Linear	Linear	9.1×10^{-3}	34.6
5	0.02	Linear	Linear	4.78×10^{-2}	157

[a]Circular transmitted and received RCS are 6 dB lower than linear transmitted and received RCS.

When these calculations are compared with data generated by the BRACT computer program (Ref. 1) for the same cases (Figs. 2 through 6), it is seen that the general Van Vleck calculations are nearly identical to those of BRACT. There is, however, a difference of at most 2 dB in the maximum RCS in the lobes between end-on (0 deg) and broadside (90 deg) for the $\beta \ell = 17$ and 157 cases. Further, the nulls for the general Van Vleck theory are much deeper than for BRACT for the larger $k\ell$ values. A comparison of the general and approximate Van Vleck theories is presented in Figs. 7 through 11. Here it is seen that, except for end-on, the results are almost identical to those of Figs. 2 through 6. Note that the approximate

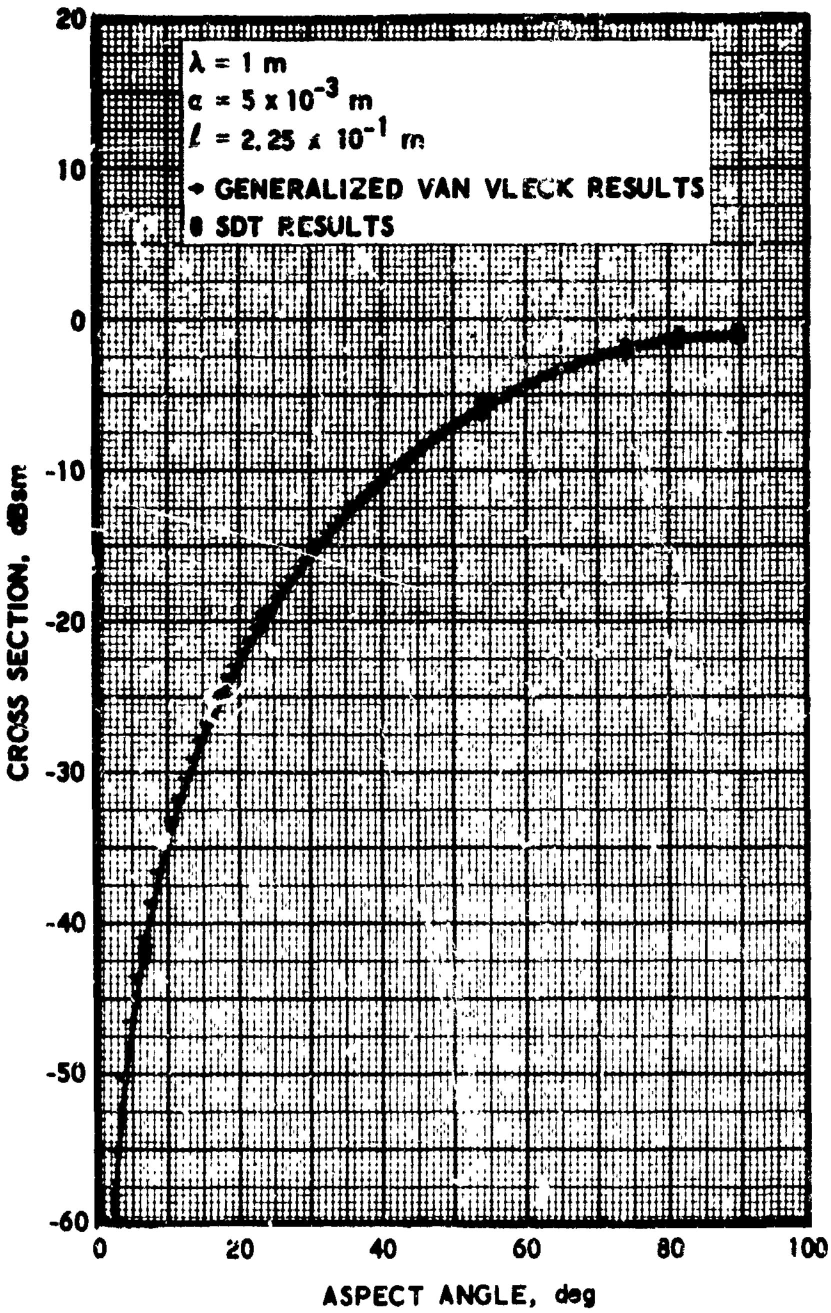

Fig. 2. Generalized Van Vleck versus SDT Results; Monostatic Cross Section of a Dipole, Linear Polarization

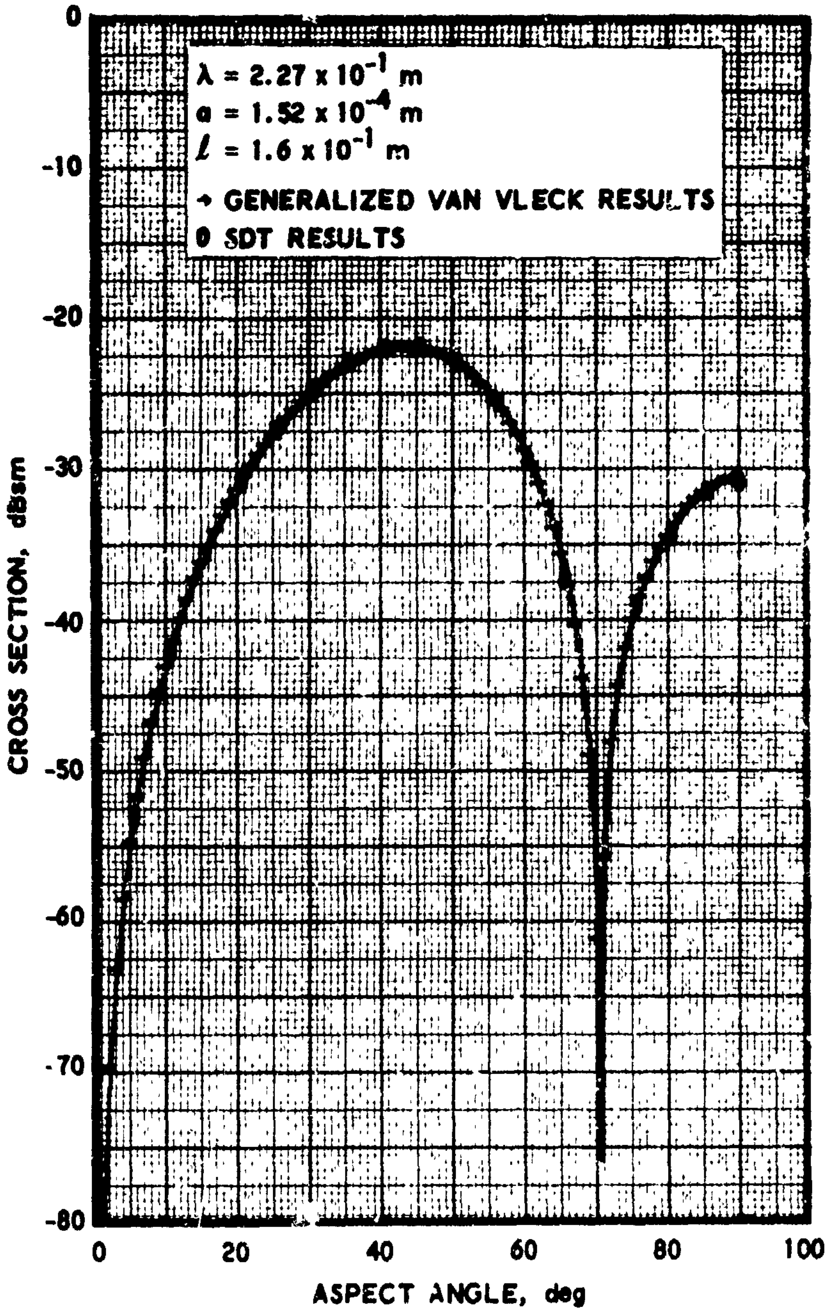

Fig. 3. Generalized Van Vleck versus SDT Results: Monostatic Cross Section of a Wire, Circular Polarization, $2\ell/\lambda = 1.4097$

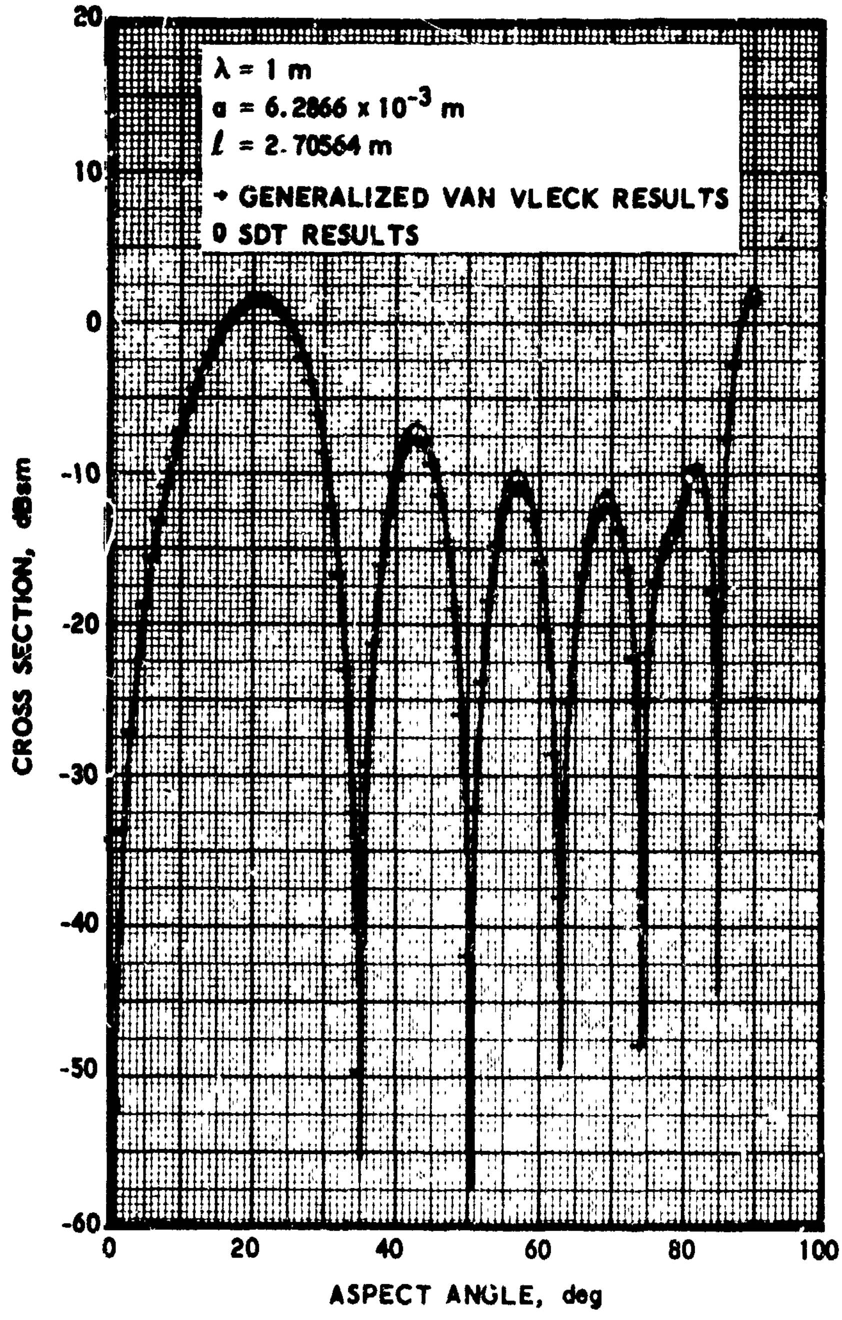

Fig. 4. Generalized Van Vleck versus SDT Results: Monostatic Cross Section of a Wire, Circular Polarization, $2\ell/\lambda$ 5.4113

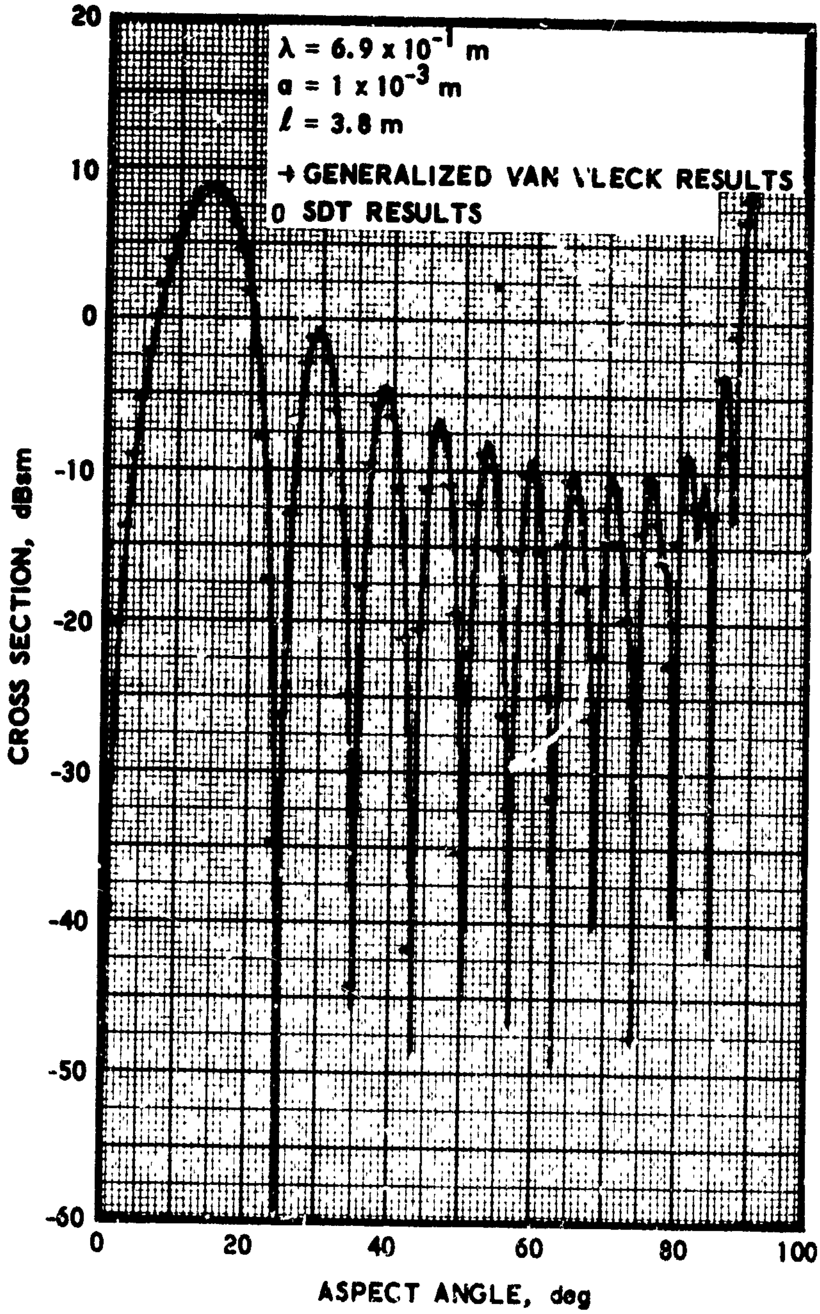

Fig. 5. Generalized Van Vleck versus SDT Results: Monostatic Cross Section of a Wire, Linear Polarization, $2\ell/\lambda = 11.0145$

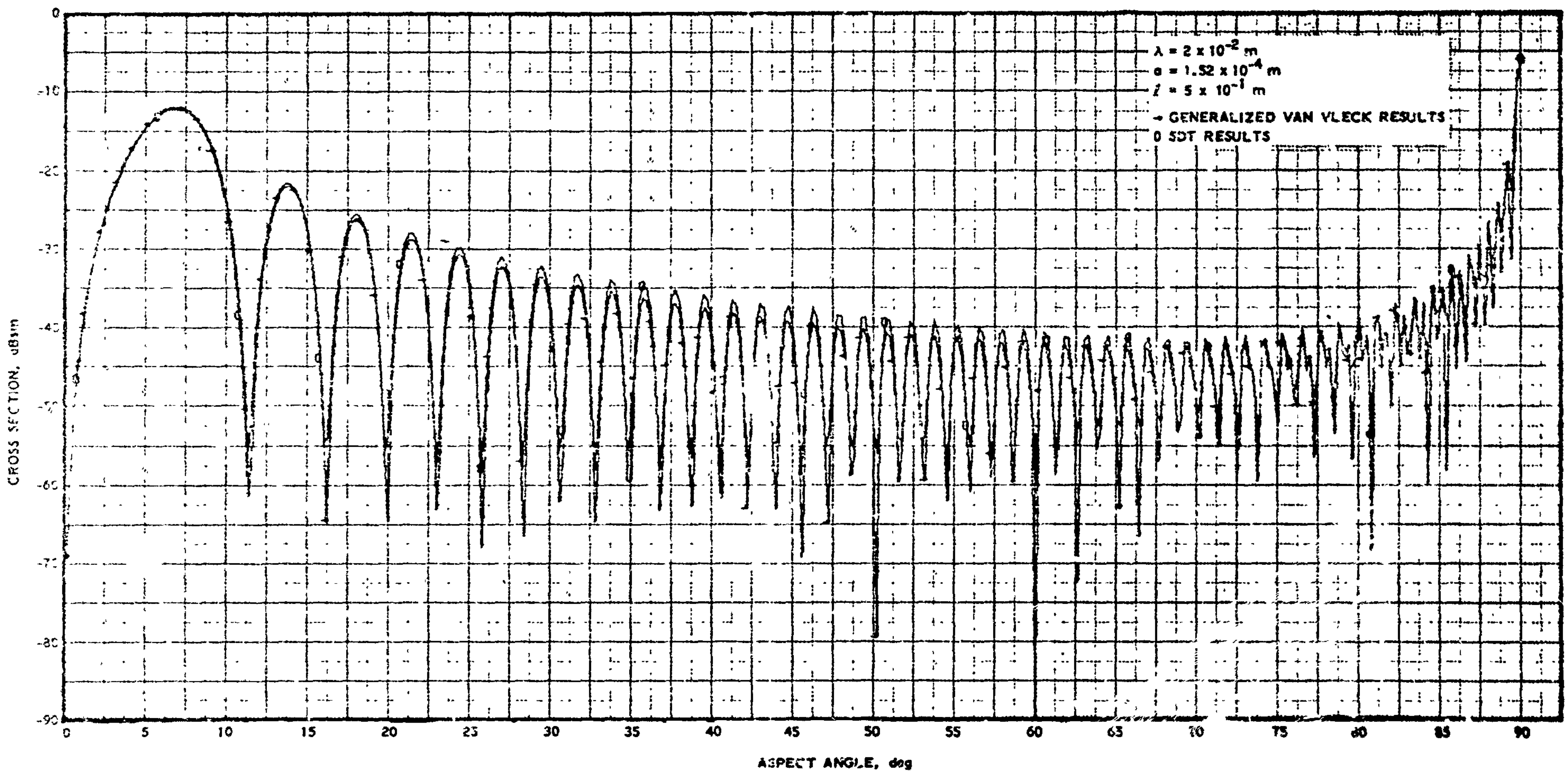

Fig. 6. Generalized Van Vleck versus SDT Results: Monostatic Cross Section of a Wire, Linear Polarization, $2\ell/\lambda = 50$

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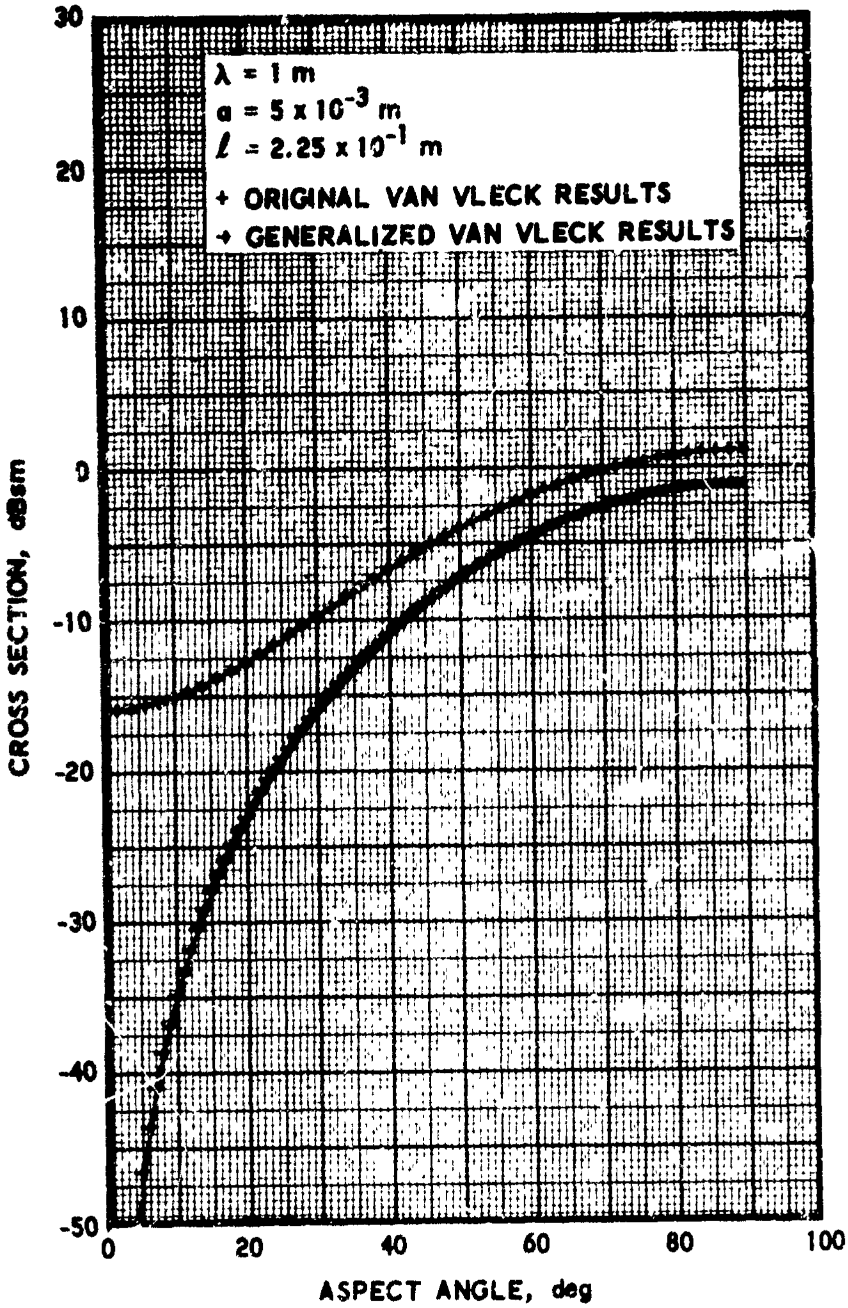

Fig. 7. Original Van Vleck versus Generalized Van Vleck Results: Monostatic Cross Section of a Dipole, Linear Polarization

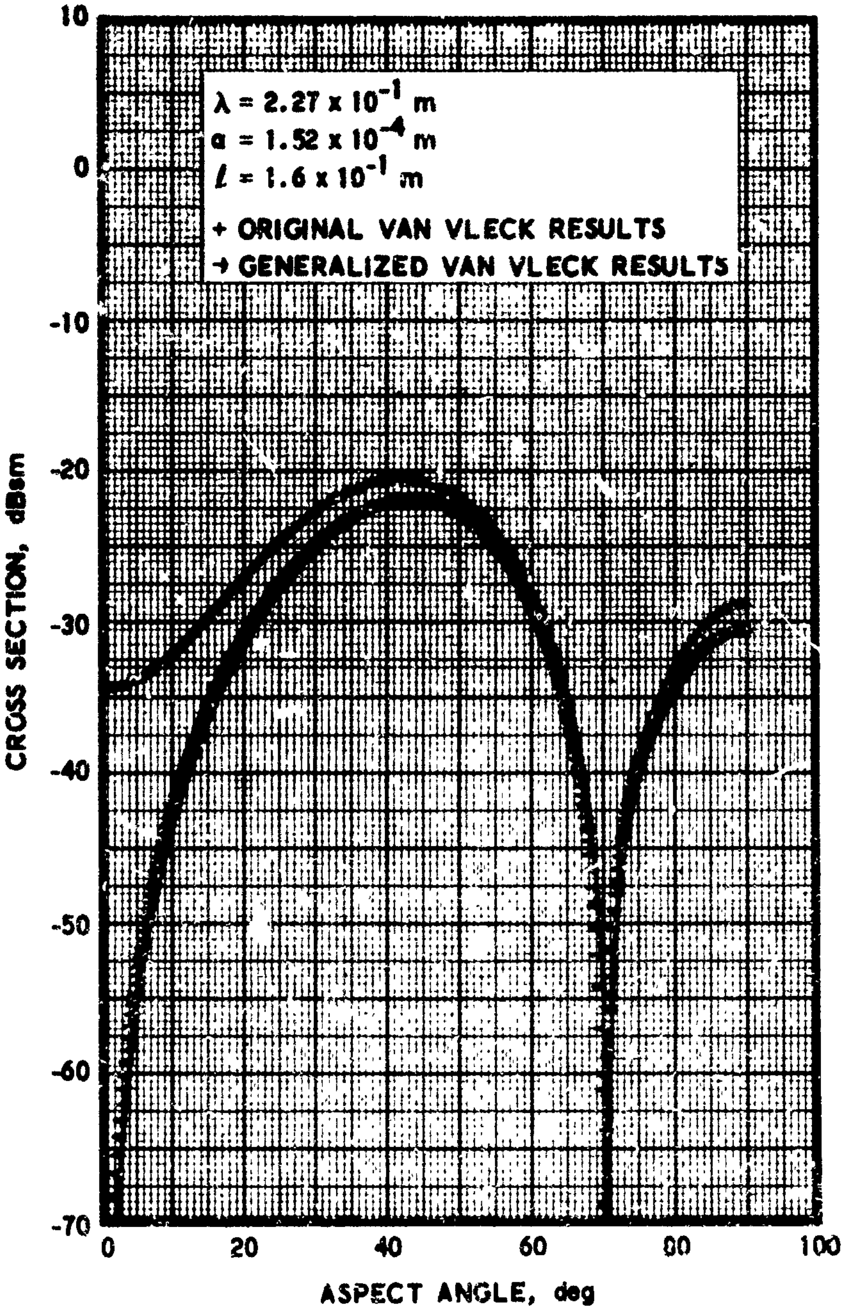

Fig. 8. Original Van Vleck versus Generalized Van Vleck Results: Monostatic Cross Section of a Wire, Circular Polarization, $2\ell/\lambda = 1.4097$

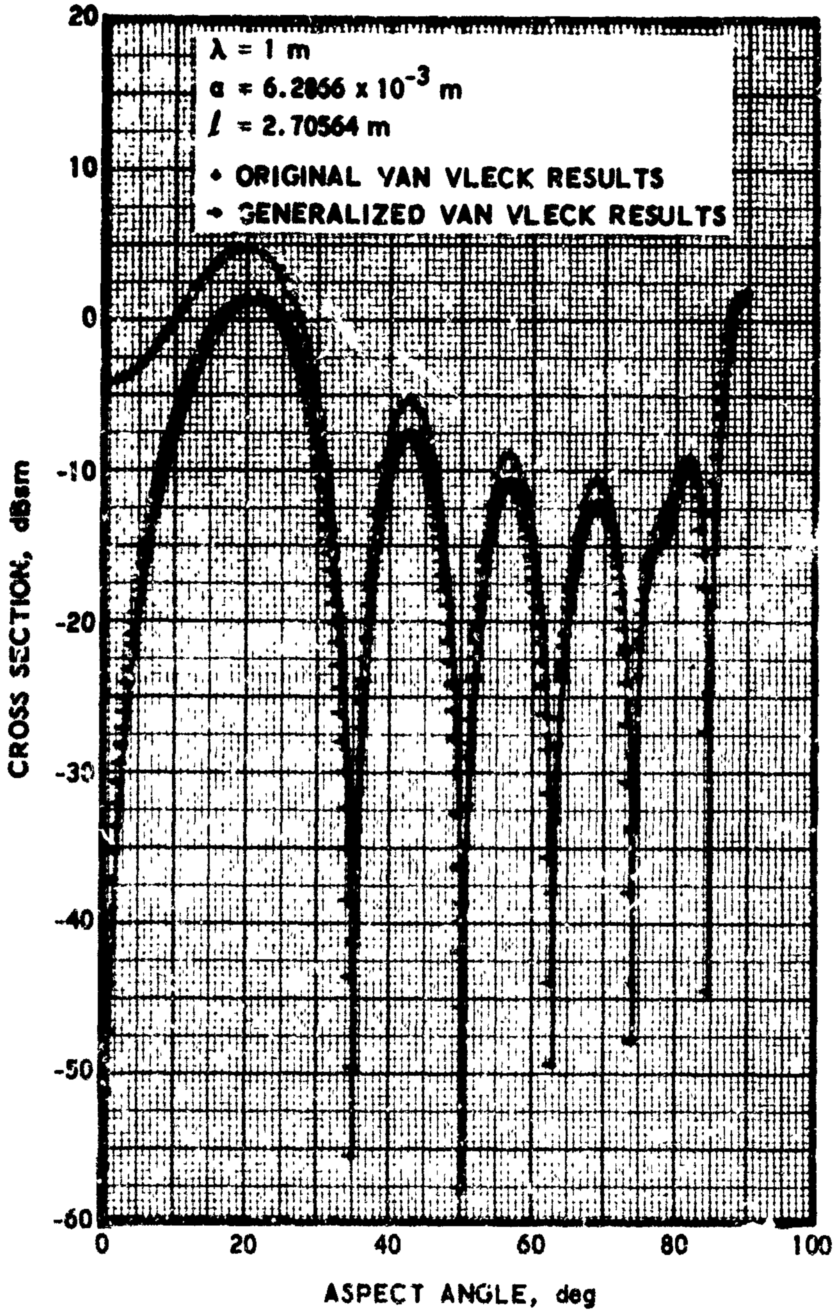

Fig. 9. Original Van Vleck versus Generalized Van Vleck Results: Monostatic Cross Section of a Wire, Circular Polarization, $2l/\lambda = 5.4113$

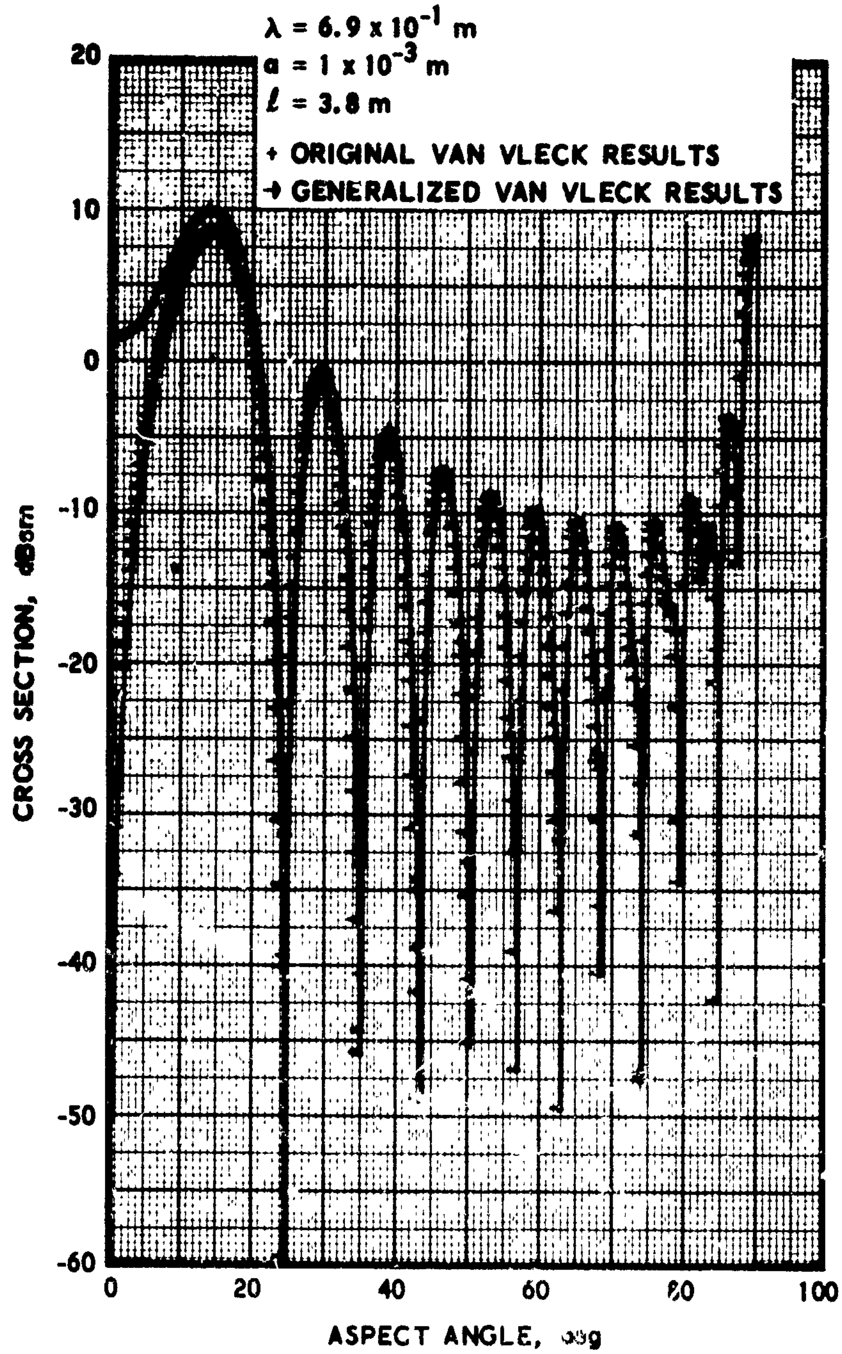

Fig. 10. Original Van Vleck versus Generalized Van Vleck Results: Monostatic Cross Section of a Wire, Linear Polarization, $2\ell/\lambda = 11.0145$

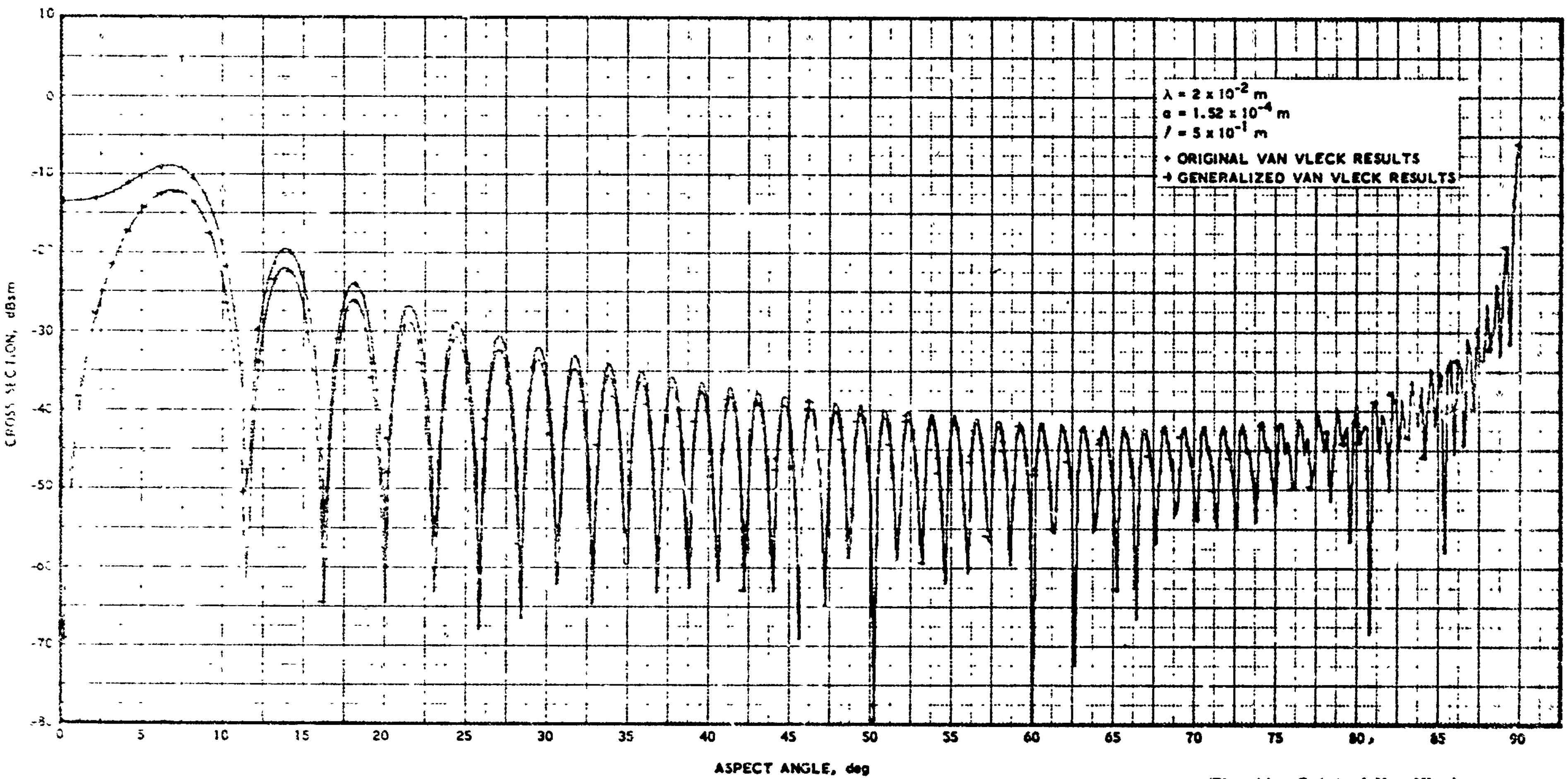

Fig. 11. Original Van Vleck versus Generalized Van Vleck Results: Monostatic Cross Section of a Wire, Linear Polarization, $2l/\lambda = 50$

Van Vleck theory agrees better with BRACT (Fig. 12) than does the general theory except for end-on incidence for larger $\beta \ell$ values. This is seen by comparing Figs. 6 and 12. For the purpose of comparison, Ufimtsev's equations have been programmed and used to calculate the RCS of the thin wire cases given in Table 1. These calculations are presented, with the BRACT calculations, in Figs. 13 through 17. It is seen that the agreement with BRACT is not as good as the agreement between the general Van Vleck and the BRACT results for $\beta \ell < 35$; for these larger $\beta \ell$ values, results obtained using Ufimtsev's equations generally agree more closely with BRACT than do the general theory results, expecially in the RCS nulls.

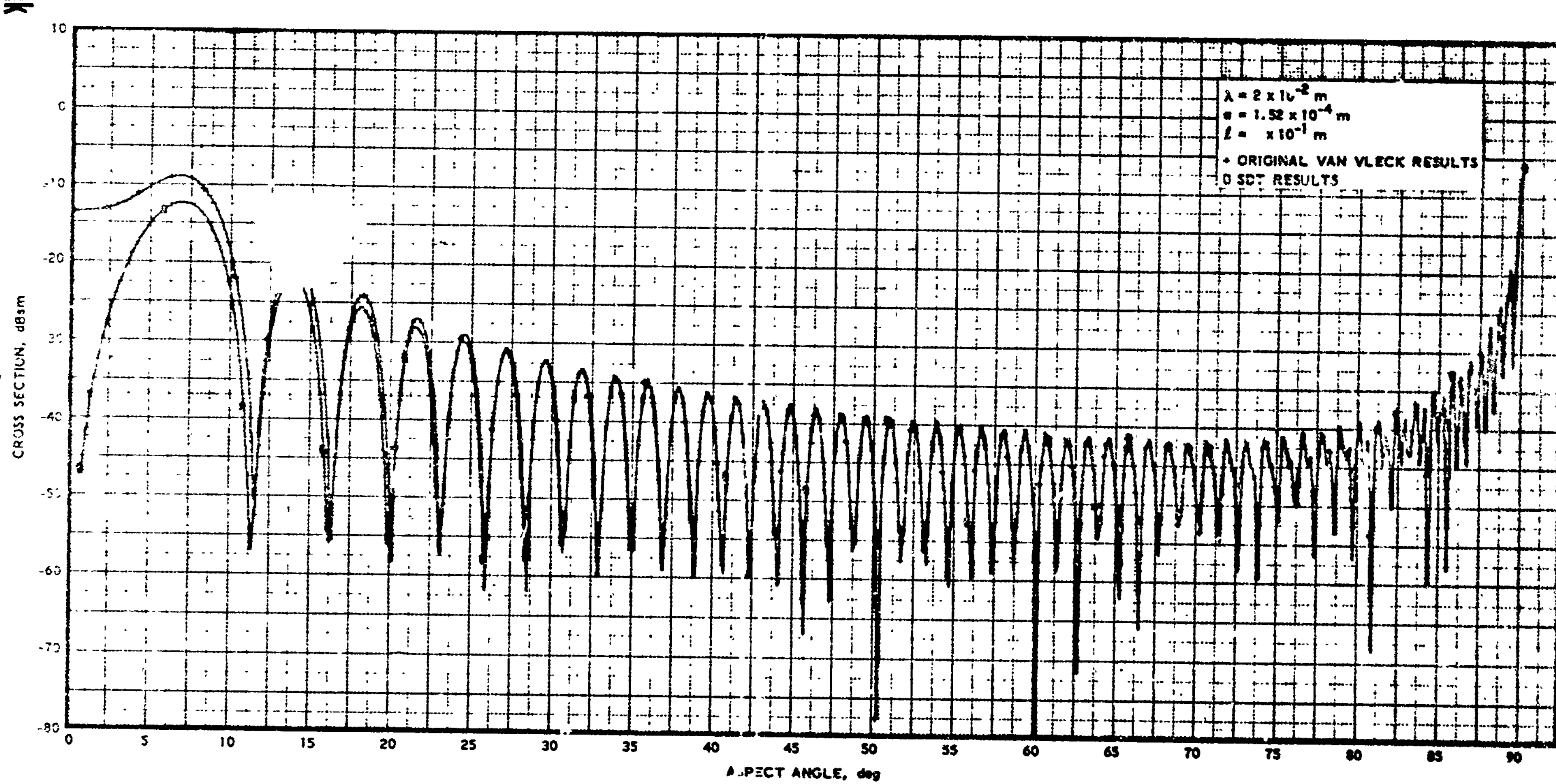

Fig. 12. Original Van Vleck versus SDT Results: Monostatic Cross Section of a Wire, Linear Polarization, $2\ell/\lambda = 50$

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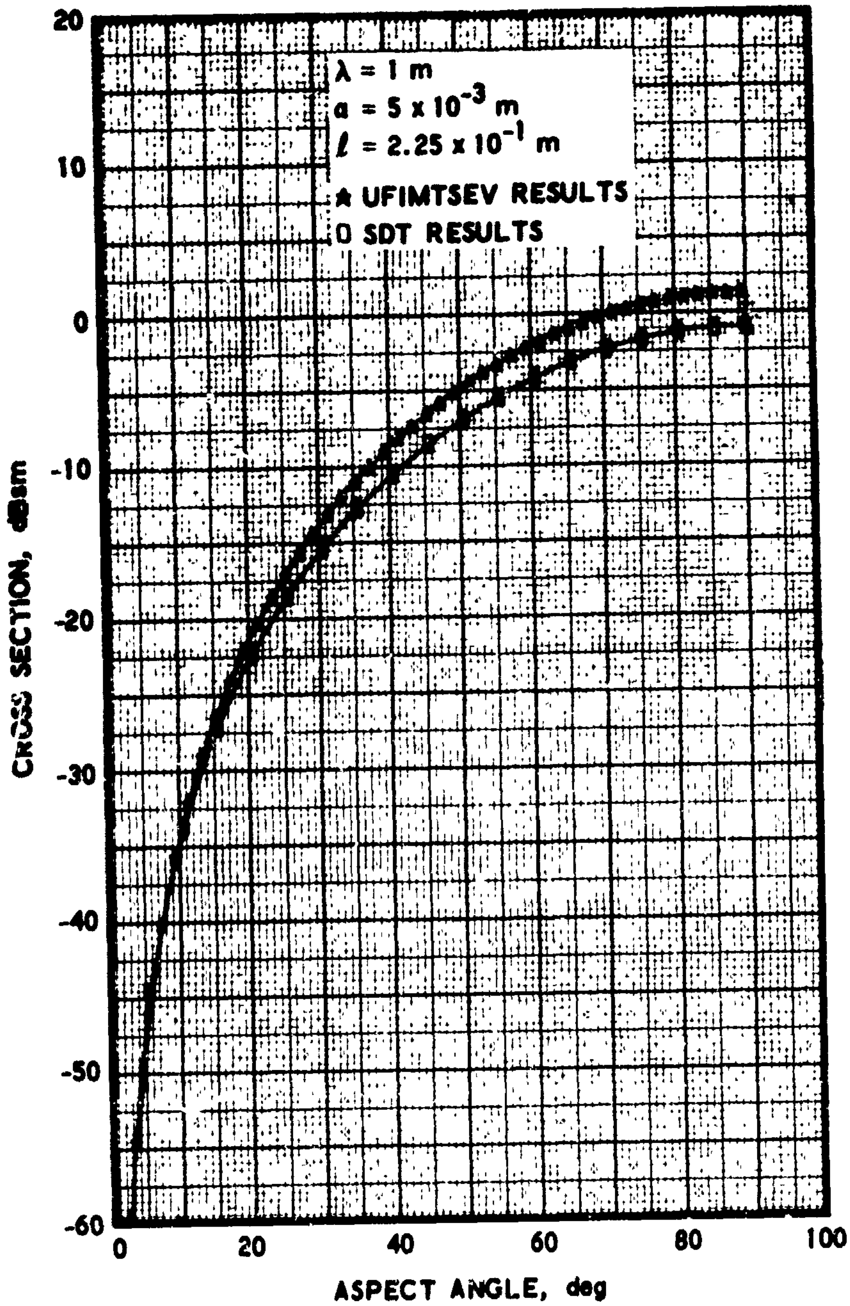

Fig. 13. Ufimtsev versus SDT Results; Monostatic Cross Section of a Dipole, Linear Polarization

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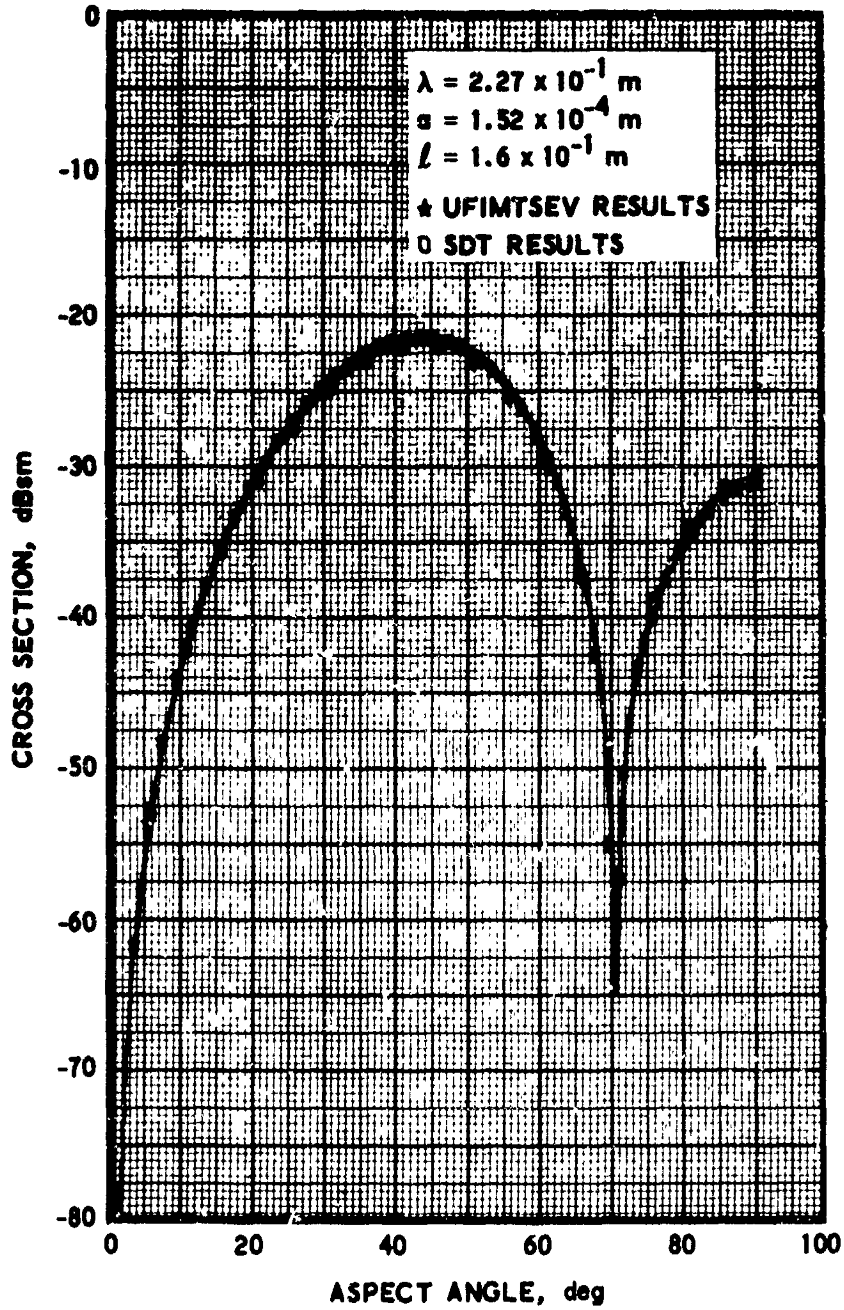

Fig. 14. Ufimtsev versus SDT Results: Monostatic Cross Section of a Wire, Circular Polarization, $2\ell/\lambda = 1.4097$

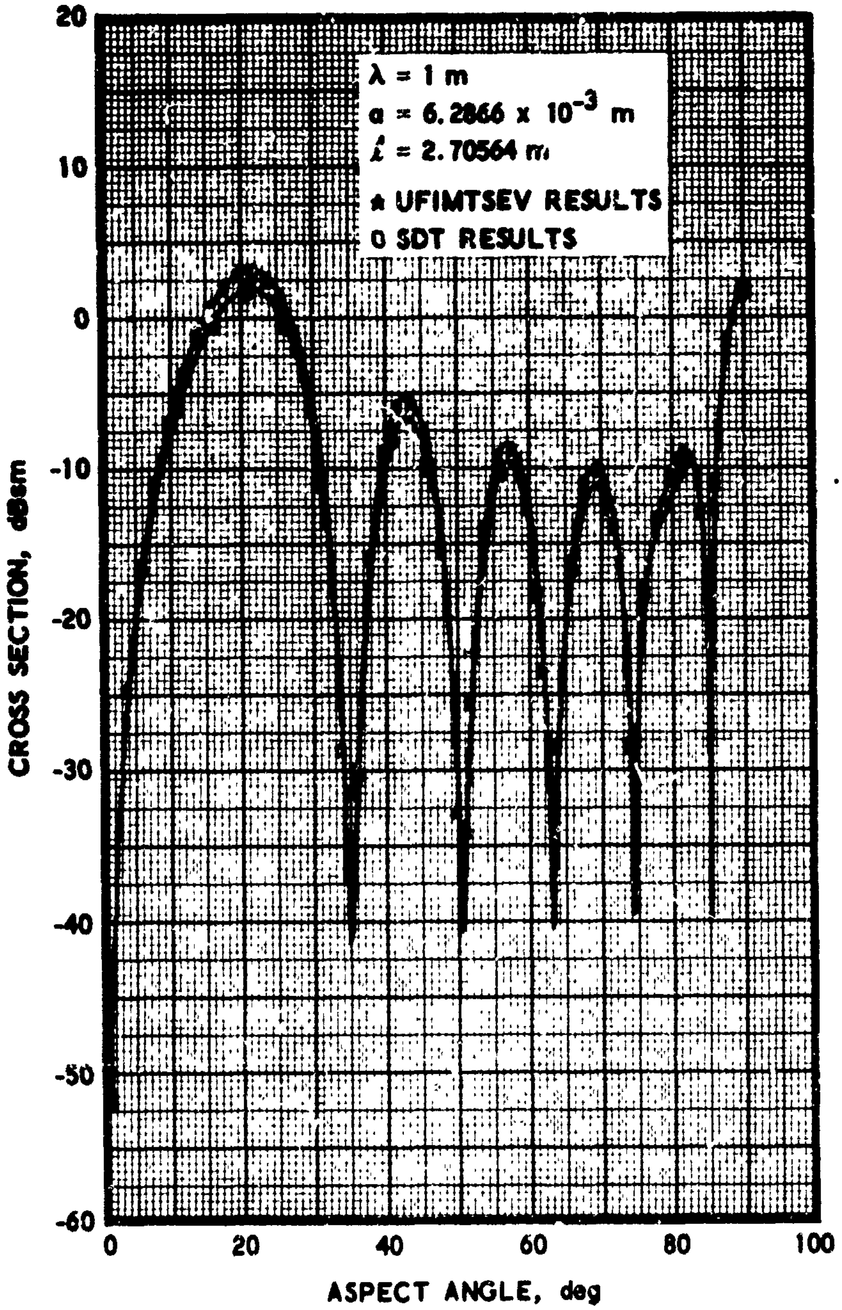

Fig. 15. Ufimtsev versus SDT Results: Monostatic Cross Section of a Wire, Circular Polarization, $2\ell/\lambda = 5.4113$

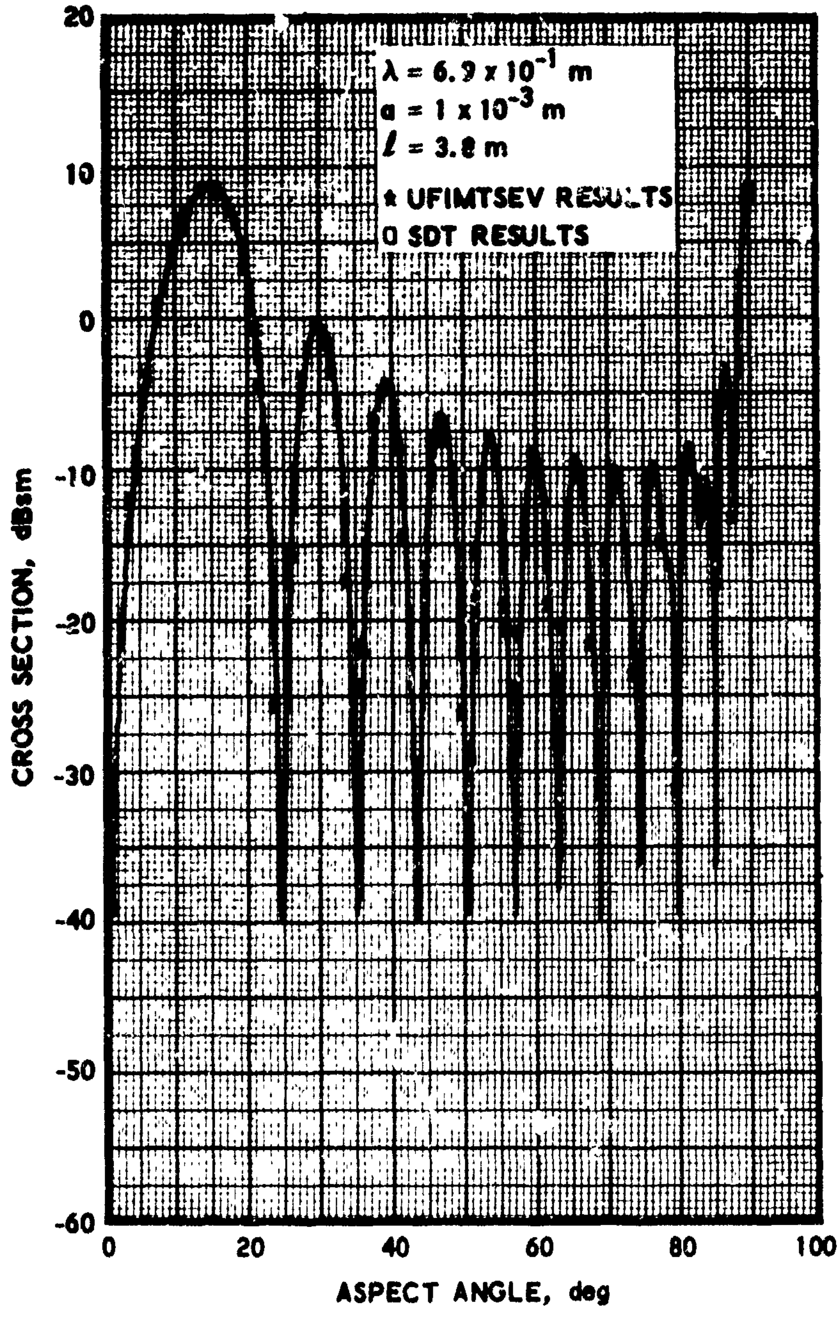

Fig. 16. Ufimtsev versus SDT Results: Monostatic Cross Section of a Wire, Linear Polarization, $2\ell/\lambda = 11.0145$

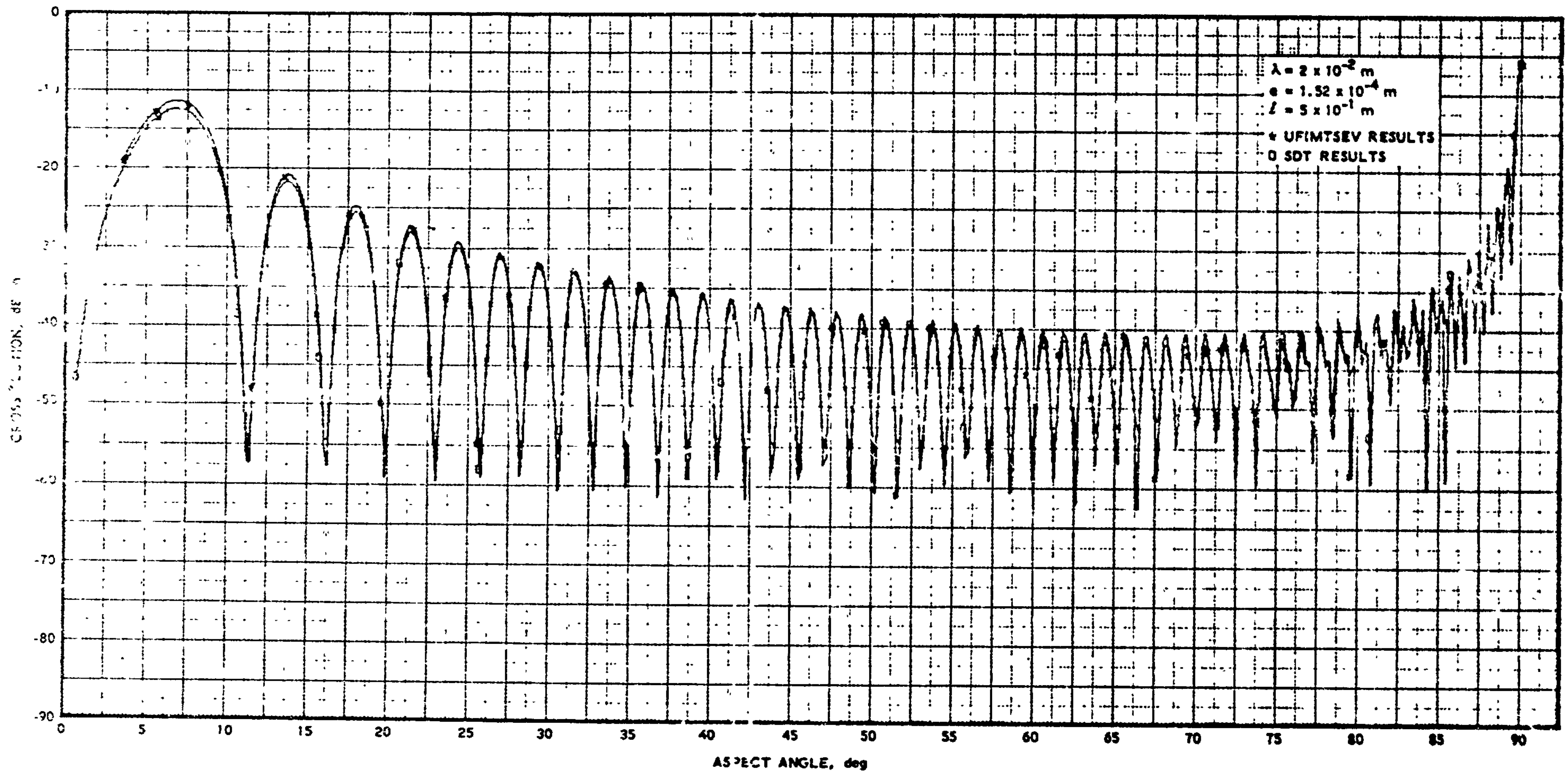

Fig. 17. Ufimtsev versus SDT Results: Monostatic Cross Section of a Wire, Linear Polarization, $2\ell/\lambda = 50$

IV. DISCUSSION

In this paper, the theory of Van Vleck, et al., has been reexamined in some detail to determine if the general results, without the approximations, are applicable for all angles of incidence. It has been found that up to $\beta \ell = 157$, the general theory agrees very well with the RCS results calculated by the BRACT computer program. At the largest $\beta \ell$ value considered ($\beta \ell = 157$), the difference is less than 2 dB at the RCS maxima, though it is considerably larger at the nulls. The disagreement is smaller at the maxima for thinner wires. Calculations have been performed to verify these findings. When results obtained using the approximate and the general Van Vleck theories are compared, it is found that differences occur that cannot be accounted for by the neglect of angular dependence in the K, G', G'', H', and H'' terms of Van Vleck, et al. These further differences are due to the "small" terms dropped from the G', G'', H', and H'' expressions and not to the use of the asymptotic expressions for the Cin and Si functions. The correct expressions with these terms are

$$2G' = \frac{\psi(\beta\ell)\left(1 - \frac{\pi}{2} F''\right) - \frac{\pi}{2} F' \,\Xi(\beta\ell)}{\psi^2(\beta\ell) + \Xi^2(\beta\ell)} \tag{36}$$

$$= \frac{\Xi(\beta\ell)\left(1 - \frac{\pi}{2} F''\right) + \frac{\pi}{2} F' \,\psi(\beta\ell)}{\psi^2(\beta\ell) + \Xi^2(\beta\ell)} \tag{37}$$

$$2H' = \frac{\psi\left(\beta\ell - \frac{\pi}{2}\right)\left(1 - \frac{\pi}{2} F''\right) - \frac{\pi}{2} F' \,\Xi\left(\beta\ell - \frac{\pi}{2}\right)}{\psi^2\left(\beta\ell - \frac{\pi}{2}\right) + \Xi^2\left(\beta\ell - \frac{\pi}{2}\right)}$$

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In this paper, the theory of Van Vleck, et al., has been reexamined in some detail to determine if the general results, without the approximations, are applicable for all angles of incidence. It has been found that up to $\beta\ell = 157$, the general theory agrees very well with the RCS results calculated by the BRACT computer program. At the largest $\beta\ell$ value considered ($\beta\ell = 157$), the difference is less than 2 dB at the RCS maxima, though it is considerably larger at the nulls. The disagreement is smaller at the maxima for thinner wires. Calculations have been performed to verify these findings. When results obtained using the approximate and the general Van Vleck theories are compared, it is found that differences occur that cannot be accounted for by the neglect of angular dependence in the K, G′, G″, H′, and H″ terms of Van Vleck, et al. These further differences are due to the "small" terms dropped from the G′, G″, H′, and H″ expressions and not to the use of the asymptotic expressions for the Cin and Si functions. The correct expressions with these terms are

$$2G' = \frac{\psi(\beta\ell)\left(1 - \frac{\pi}{2} F''\right) - \frac{\pi}{2} F' \, \Xi(\beta\ell)}{\psi^2(\beta\ell) + \Xi^2(\beta\ell)} \tag{36}$$

$$= \frac{\Xi(\beta\ell)\left(1 - \frac{\pi}{2} F''\right) + \frac{\pi}{2} F' \, \psi(\beta\ell)}{\psi^2(\beta\ell) + \Xi^2(\beta\ell)} \tag{37}$$

$$2H' = \frac{\psi\left(\beta\ell - \frac{\pi}{2}\right)\left(1 - \frac{\pi}{2} F''\right) - \frac{\pi}{2} F' \, \Xi\left(\beta\ell - \frac{\pi}{2}\right)}{\psi^2\left(\beta\ell - \frac{\pi}{2}\right) + \Xi^2\left(\beta\ell - \frac{\pi}{2}\right)}$$

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